

## **A discussion of yarn measures and how they relate to yarn diameters, Sett, EpI and PpI, Warp Crimp and fabric weight.**

*Disclaimer:* I am not a weaver. As the author of the JacqCAD program I've enjoyed a lot of contact with weavers and have had to research a goodly range of textile topics, but my knowledge is theoretical rather than experience based. As long as you keep alert for errors and misunderstandings, you may find some of the following information useful, or at least thought provoking. Corrections are actively solicited!

Yarn densities - "Sett", Ends per Inch (EpI) and Picks per Inch (PpI) - are based on yarn diameters, but yarn diameters are not part of the standard yarn specification, they have to be calculated. Information about this step is scattered, often in the form of "rules of thumb" whose assumptions are often poorly defined and which often raise more questions than they answer.

A classic example is the century-old rule of thumb "take the square root of YpP and subtract 7% to get Sett". Why 7%, why not 5% or 10%? What if I'm using acrylic rather than cotton? What if the yarn is a tight twist versus a loose twist? The rule gives no guidance for such reasonable questions.

So I've tried to collate a set of well defined rules for converting between the many yarn measure systems, calculating yarn diameters, and estimating maximum EpI/PpI, Cover, Warp Crimp, and Fabric Weight.

I've divided the discussion into the following topics:

**Yarn measures** - converting between the variety of systems

**Yarn diameter** - calculating diameter from yarn measure, material and twist

**Yarn spacing** - using yarn diameters to estimate minimum spacings: Sett, EpI, PpI

**Cover** - how much of the fabric is yarns versus gaps between yarns

**Shrinkage** - fabric shrinks in both directions when it is removed from the loom and finished

**Warp crimp** - the warp used is longer than the fabric woven

**Fabric weight** - weight per square yard

## YARN MEASURES:

All yarn measure systems are just ratios between the weight and length of a sample of yarn

**Direct** systems = weight / fixed length; bigger number = bigger yarn

**Indirect** systems = length / fixed weight; bigger number = smaller yarn

The units of measure used include: grams or pounds for weight, and “hanks” of various lengths, yards, meters, or km for length.

Common direct systems:

**Tex** = Tex                      g / 1 km  
**dTex** = deciTex                g / 10 km  
**Den** = Denier                 g / 9 km    {origin: 1 silk fiber  $\approx$  1 Den}

Common indirect systems:

**NeC** = Cotton count            #840 Yd hanks/ Lb    {sometimes shown as just **Ne** }  
**NeK** = Worsted count        #560 Yd hanks/ Lb  
**NeL** = Linen count            #300 Yd hanks/ Lb  
**NeW** = Wool count            #256 Yd hanks/ Lb  
**Nm** = Metric count            km / kg  
**YpP** = yield                    Yds / Lb

**T1: TO CONVERT WITHIN similar Yarn Measure Systems MULTIPLY “from” value by TABLE ENTRY:**

<i>from:</i>	<i>NeC</i>	<i>NeK</i>	<i>NeL</i>	<i>NeW</i>	<i>Nm</i>	<i>YpP</i>
<b>TO: NeC</b>	1	0.6667	0.3571	0.3048	0.5905	0.001,190
<b>NeK</b>	1.50	1	0.5357	0.4571	0.8858	0.001,786
<b>NeL</b>	2.80	1.8667	1	0.8533	1.6535	0.003,333
<b>NeW</b>	3.281	2.188	1.1717	1	1.9376	0.003,905
<b>Nm</b>	1.693	1.1289	0.6048	0.5161	1	0.002,016
<b>YpP</b>	840	560	300	256	496.06	1

<i>from:</i>	<i>Den</i>	<i>Tex</i>	<i>dTex</i>
<b>TO: Den</b>	1	9	0.9
<b>Tex</b>	0.1111	1	0.1
<b>dTex</b>	1.111	10	1

Note: Table Entries are  
**MULTIPLIERS**  
 new = table entry **X** old (from) value

**T2: TO CONVERT BETWEEN Yarn Measure Systems DIVIDE “from” Value INTO THE TABLE ENTRY**

<i>from:</i>	<i>Den</i>	<i>Tex</i>	<i>dTex</i>	<i>NeC</i>	<i>NeK</i>	<i>NeL</i>	<i>NeW</i>	<i>Nm</i>	<i>YpP</i>
<b>TO: Den</b>				5,315	7,972	14,882	17,439	9,000	4,464,490
<b>Tex</b>	Use Table T1			590.5	885.8	1,654	1,938	1,000	496,055
<b>dTex</b>				5,905	8,858	16,535	19,377	10,000	4,960,550
<b>NeC</b>	5,315	590.5	5,905						
<b>NeK</b>	7,972	885.8	8,858						
<b>NeL</b>	14,882	1,654	16,535			Use Table T1			
<b>NeW</b>	17,439	1,938	19,377						
<b>Nm</b>	9,000	1,000	10,000						
<b>YpP</b>	4,464,490	496,055	4,960,550						

new “to” measure = TABLE entry DIVIDED BY “from” measure value

Example : given a #10 NeC, what is that in YpP? (both are indirect systems)

from Table **T1**: FROM NeC TO YpP: 840 MULTIPLIER, so multiply  $840 \times 10 \text{ NeC} = \mathbf{8,400 \text{ YpP}}$

what is 8,400 YpP in Tex? (converting from indirect to direct)

from Table **T2**: FROM YpP TO Tex: 496,055 TO BE DIVIDED INTO so  $496,055 / 8,400 \text{ YpP} = \mathbf{59.05 \text{ Tex}}$

closing the circle: what is 59.05 Tex in NeC? (converting from direct to indirect)

from Table **T2**: FROM Tex TO Nec: 590.5 TO BE DIVIDED INTO so  $590.5 / 59.05 = \mathbf{10 \text{ NeC}}$

## YARN DIAMETERS:

To figure out “sett”, EpI or PpI we need to know the yarn diameter.

**Diam** = diameter of yarn in inches

**YpI\_100** = Yarns per Inch at 100% coverage, i.e. wrapped side-by-side so they are just touching without squashing. In the literature this is sometimes referred to as “Diameters”, note the plural and read carefully!

**YpI\_100** = 1/ **Diam** these are simply reciprocals of each other, 50 YpI\_100 = 1/50 Diameter & vice-versa

However, the yarn measures relate only length and weight. Manufacturers find it easy to weigh a fixed length of yarn, or measure the length of a fixed weight of yarn, but much more difficult to define and measure the diameter of a soft fluffy yarn.

To illustrate the problem, this table shows the diameter, YpI\_100 and the weight (ounces / square yard) of yarn at 100% coverage for various yarns, **every one of which is a 500 denier yarn**:

YpI_100	Diam “	Oz/SqY	Material
64	0.0156	4.1	Polypropylene fiber, medium twist (50% Fiber Fraction FF)
77	0.0129	5.0	Wool, medium twist (50% FF)
83	0.0120	5.4	Cotton, medium twist (50% FF)
91	0.0110	5.8	Polypropylene monofilament (100% FF)
152	0.0066	9.8	Glass, monofilament (100% FF)
309	0.0032	20.0	Silver wire (100% FF)
420	0.0024	27.1	Gold wire (100% FF)

**As you can see above, yarn measure is only a starting point!**

Yarn diameter (and YpI\_100) are related to Yarn Measure by the Yarn Density, **Ydens**, which is the density of the yarn *including* the air between the fibers

**Ydens = Fiber Fraction X Fiber Density**, where:

**T3: Fiber Fraction** = proportion of yarn that is actual fiber, not counting the included air. It depends largely on the amount of twist. Some typical values:

- 1.00 monofilament (single fiber, no air spaces)
- 0.90 theoretical maximum by perfect packing of identical round fibers
- 0.80 extra firm twist
- 0.60 regular warp twist 60% fiber 40% air - warps
- 0.50 medium soft spun yarn - wefts
- 0.35 open lofty soft twist yarn - knitting yarns

**T4: Fiber Density** = density (g/cc) of the solid material, e.g., wool. Typical values:

Natural, vegetable fibers	Synthetics - Polymers
1.52 Cotton	1.71 Saran
1.51 Ramie	1.41 Aramid
1.50 Jute	1.38 Terylene
1.50 Flax (linen)	1.33 Modacrylic
1.48 Hemp	1.29 Polyester
Natural, animal fibers	1.18 Polyacrylonitrile
1.31 Wool,	1.17 Acrylic
1.30 Silk, raw	1.14 Nylon
Synthetics - Cellulosic	1.03 Qiana nylon
1.52 Rayon	0.90 Polypropylene olefin
1.32 Cellulose Acetate	
1.30 Cellulose triacetate	
Miscellany	
19.3 Gold	1.000 water (all others relative to water)
10.5 Silver	
2.54 Glass	

## CONVERTING Yarn Measure + Yarn Density (Ydens) to Diameters or YpI\_100

Weight is directly proportional to area, which in turn is proportional to the square of the diameter. Doubling the direct yarn measure or Ydens doubles weight, but doubling the diameter quadruples weight. Going the other direction, diameter will be proportional to the *square root* of area and density.

Given:

**mDir** = some yarn measure using a direct system, e.g., Tex, deciTex, Den, or

**mInd** = some yarn measure using an indirect system, e.g., NeC, NeK, NeL, NeW, Nm or YpP

$$1) \text{ Diam} = K1_{dir} / \text{sqrt}[ \text{Ydens} / \text{mDir} ] = K1_{ind} / \text{sqrt}[ \text{Ydens} \times \text{mInd} ]$$

$$2) \text{ YpI}_{100} = K2_{dir} \times \text{sqrt}[ \text{Ydens} / \text{mDir} ] = K2_{ind} \times \text{sqrt}[ \text{Ydens} \times \text{mInd} ]$$

where:

**sqrt()** indicates the square root of whatever is inside the (), and

**K1\_xxx** is a scaling factor specific to the yarn measure system being used and to the units desired for **Diam** and **YpI\_100** (see Table T5)

$$K2_{xxx} = 1 / K1_{xxx}$$

**T5: K1 and K2 FOR RESULTS (Yarn Diameter or YpI) IN INCHES :**

		Yarn Measure System	
<b>K1<sub>dir</sub></b>	<b>K2<sub>dir</sub></b>	<b>mDir:</b> direct systems	
0.001,405	711.8	<b>Tex</b> = Tex	g / 1 km
0.000,444	2,251	<b>dTex</b> = deciTex	g / 10 km
0.000,468	2,136	<b>Den</b> = Denier	g / 9 km
<b>K1<sub>ind</sub></b>	<b>K2<sub>ind</sub></b>	<b>mInd:</b> indirect systems	
0.034,1	29.29	<b>NeC</b> = Cotton count	#840 Yd hanks/ Lb
0.041,8	23.92	<b>NeK</b> = Worsted count	#560 Yd hanks/ Lb
0.057,1	17.51	<b>NeL</b> = Linen count	#300 Yd hanks/ Lb
0.061,8	16.17	<b>NeW</b> = Wool count	#256 Yd hanks/ Lb
0.044,4	22.51	<b>Nm</b> = Metric count	km / kg
0.989	1.011	<b>YpP</b> = yield	Yds / Lb

Examples:

500 denier wool (wool density = 1.31), med warp twist (Fiber Fraction **FF** = 0.60)

Ydens = 0.60 FF x 1.31 wool density

**Ydens** = 0.786

$$\begin{aligned} \mathbf{YpI}_{100} &= \mathbf{K2}_{dir} \times \text{sqrt}(\mathbf{Ydens} / \mathbf{mDir}) \\ &= 2136 \times \text{sqrt}(0.786 / 500 \text{ den}) = \mathbf{84.69 YpI}_{100} \end{aligned}$$

same yarn using different direct system yarn measures

$$55.55 \mathbf{Tex} = 711.8 \text{sqrt}(0.786 / 55.55 \mathbf{Tex}) = 84.67 \mathbf{YpI}_{100}$$

$$555.5 \mathbf{dTex} = 2251 \text{sqrt}(0.786 / 555.5 \mathbf{dTex}) = 84.67 \mathbf{YpI}_{100}$$

same yarn using indirect system yarn measures :  $\mathbf{YpI}_{100} = \mathbf{K2}_{ind} \text{sqrt}(\mathbf{mInd} \times \mathbf{Ydens})$

$$10.63 \mathbf{NeC} = 29.29 \text{sqrt}(10.63 \mathbf{NeC} \times 0.786) = 84.66 \mathbf{YpI}_{100}$$

$$15.94 \mathbf{NeK} = 23.92 \text{sqrt}(15.94 \mathbf{NeK} \times 0.786) = 84.67 \mathbf{YpI}_{100}$$

$$29.76 \mathbf{NeL} = 17.51 \text{sqrt}(29.76 \mathbf{NeL} \times 0.786) = 84.69 \mathbf{YpI}_{100}$$

$$18.00 \mathbf{Nm} = 22.51 \text{sqrt}(18.00 \mathbf{Nm} \times 0.786) = 84.67 \mathbf{YpI}_{100}$$

$$8929 \mathbf{YpP} = 1.011 \text{sqrt}(8929 \mathbf{YpP} \times 0.786) = 84.70 \mathbf{YpI}_{100}$$

which confirms that the K2 constants are consistent (within the limits of rounding errors).

The K1 values correspond to the diameter *in inches* of a #1 monofilament yarn whose density = 1.000 and the K2 values correspond to the YpI<sub>100</sub> of the same yarn.

For example, the diameter of a #1 NeC monofilament of density 1.000 would be 0.0341 inches (K1<sub>ind</sub> for NeC) and a YpI<sub>100</sub> of 29.29 (K2<sub>ind</sub> for NeC), while the diameter of a #1 Den monofilament would be 0.000468 inches (K1<sub>dir</sub> for Den) and a YpI<sub>100</sub> of 2136 (K2<sub>dir</sub> for Den).

The equations on page 4 simply scale the diameters and YpI<sub>100</sub> for yarn measures other than #1 and for yarn densities other than 1.000.

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Thanks to Jacqueline Mohen of Philadelphia University for identifying a need for improved clarity - which was achieved by changing the format of one equation and splitting the K1 (and K2) notation into separate K1<sub>dir</sub> and K1<sub>ind</sub> constants.

**Derivation** (for the mathematically inclined):

Weight of yarn per cm length =  $W_{cm}$  = volume x density = area x length x density

$$\begin{aligned}W_{cm} &= \pi \times R^2 \text{ in } cm^2 \times 1 \text{ cm} \times Y_{dens} \text{ in } g/cm^3 \quad \text{units are: } cm^2 \times cm \times g/cm^3 \text{ per cm} = g/cm \\ &= \pi \times (D/2)^2 \times 1 \times Y_{dens} \quad g/cm \\ &= \pi \times (D)^2/4 \times Y_{dens} \quad g/cm \\ &= 0.25 \times \pi \times (D)^2 \times Y_{dens} \quad g/cm\end{aligned}$$

Tex = weight in grams of 1 km = weight in grams of 100,000 cm of yarn = 100,000 x  $W_{cm}$

$$\text{Tex} = 100,000 \times 0.25 \times \pi \times D^2 \times Y_{dens} = 25,000 \times \pi \times D^2 \times Y_{dens} \quad g/km$$

solving for D (noted as  $D_{cm}$  to remind us we are working in cm):

$$D_{cm}^2 = \text{Tex} / (25,000 \times \pi \times Y_{dens}) = (1/(25,000 \times \pi)) \times (\text{Tex} / Y_{dens})$$

$$D_{cm} = \text{sqrt}[ (1/(25,000 \times \pi)) ] \times \text{sqrt}[ (\text{Tex} / Y_{dens}) ] \quad cm$$

$$D_{cm} = 0.003568 \times \text{sqrt}[ (\text{Tex} / Y_{dens}) ] \quad cm$$

$$D_{in} = D \text{ in inches} = D_{cm} / 2.54 = 0.003568/2.54 \times \text{sqrt}[ (\text{Tex} / Y_{dens}) ] \quad inch$$

$$D_{in} = 0.001405 \times \text{sqrt}[ (\text{Tex} / Y_{dens}) ] \quad inch \quad \text{which matches Table T5's K1 value for Tex. QED.}$$

**Caution about plied yarns:** Be careful about how you convert plied yarns into single strand equivalents.

Typically a 10/2 NeC indicates that 2 #10NeC strands have been plied together. The resulting yarn will be 2X as heavy as #10 NeC, which is the equivalent of a single #5 NeC (smaller indirect counts = heavier yarns).

A 60x2 Tex usually indicates 2 strands of 60 Tex each have been plied together producing a yarn that is 2X as heavy = 120 Tex single. (larger direct counts = heavier yarns).

Unfortunately there are so many systems and notations that confusion is common. Some standards interpret 60x2 Tex as being a 60 Tex equivalent single thread composed of 2 strands (each of 30 Tex) while others interpret it as a 120 Tex single thread equivalent composed of 2 60 Tex strands. It is always safest to spell it out in detail, e.g., “10/2 Nec (5 NeC single thread equivalent)” or “60x2 Tex (120 Tex single thread equivalent)”.

Specifying only a “10/2 yarn” is a recipe for disaster. You need to specify the measuring system used and the exact meaning of the plies along with material, twist and twist direction, etc.

## YARN SPACING: “Sett”, EpI and PpI

Given a particular yarn material, twist (Fiber Fraction) and yarn measure, we now know how to calculate its diameter. Its **YpI\_100** is the number of those yarns which will fit side-by-side across 1”, just touching without compression, thus producing 100% coverage. A yarn’s **YpI\_100** is simply the reciprocal of its diameter; for example, if the YpI\_100 is 50, then that yarn’s diameter is 1/50”, and vice-versa.

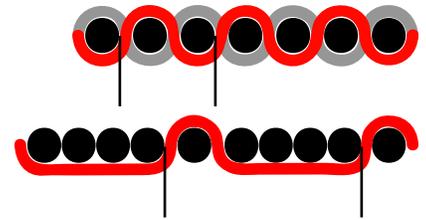
Weavers often measure **YpI\_100** by carefully wrapping a yarn around a cylinder, a rather tricky and subjective task, and then counting the number of turns per inch. Here we calculate it directly as described previously.

A yarn’s **YpI\_100** or Diameter are the starting point for calculating the appropriate yarn spacings, whether for the warp (EpI or Sett) or weft (PpI).

We want to figure out the minimum yarn spacing permitted by the yarns and weaves. That calculated spacing will produce a tight fabric that is just on the edge of compressing yarns. That produces a rather “boardy” fabric, and many fabrics will be woven at some wider spacing - for instance sheeting or table cloth will usually be woven at close to the minimum spacing while scarves would be woven and a considerably wider spacing.

The weaves being used play a crucial role.

Consider the figures to the right in which the black circles are the wefts and the red and gray are warps; the two black lines mark out one repeat of the weave.



The first shows a plain weave - each repeat must be wide enough to accommodate 2 wefts plus 2 warp piercings (the “piercings” are always an even number since the yarn must end on the same side as it began).

The second shows a 5-shaft satin - in this case each repeat must accommodate 5 wefts + 2 warp piercings .

This concept of “leave enough space for both wefts and warp piercings” is the basis for the “*Ashenhurst Rule (first version)*” and similar rules. It applies both to weft spacing (PpI) as we have been discussing, and to warp spacing (“Sett” or EpI); the only difference being that the role of the warp and weft are reversed. Rather than duplicate the discussion I will use a single generic approach which speaks about “the yarn system we are analyzing” - which will be the wefts when calculating PpI and the warps when calculating EpI or Sett.

A repeat of any weave we are analyzing will contain:

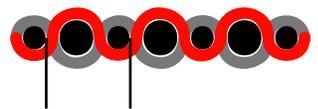
**Ny** = # of yarns in the system we are analyzing, e.g., in the weft, each of diameter **Dy**

**Np** = # of “piercings” (always an even #) by a yarn in the other system, e.g., in the warp, diameter **Dp**

The minimum space required for that repeat will be:

$$L = N_y \times D_y + N_p \times D_p, \text{ where } D_y \text{ and } D_p = \text{diameters of the two yarns.}$$

Fabrics can contain several kinds of wefts - for example this figure shows a plain weave being woven using alternating thin and thick wefts.



The only change is that now we have two different weft diameters

**Ny1** = # thin wefts in the repeat, diameter **Dy1**

**Ny2** = # thick wefts in the repeat, diameter **Dy2**

and the minimum space for that repeat will be:

$$L = N_{y1} \times D_{y1} + N_{y2} \times D_{y2} + N_p \times D_p, \text{ where } D_{y1}, D_{y2} \text{ and } D_p \text{ are the diameters of the yarns.}$$

This approach can easily be extended to cover any combinations of yarn sizes.

Note that we are analyzing the space required by a repeat along a single weft or warp. In most weaves each warp shows the same pattern, just shifted (as in a twill or satin), and the same is true along each weft. However, it certainly is possible to have weaves with different patterns. In such cases it is necessary to evaluate the different patterns, identify the one that requires the *longest length* and use that one as the determining length.

Sometimes it is easier to work with a yarn's **YpI\_100** instead of its diameter. Since **D** and **YpI\_100** are reciprocals the above equation simply becomes:

$$L = N_{y1} / Y_{pI\_100\_1} + N_{y2} / Y_{pI\_100\_2} + N_p / Y_{pI\_100\_p}$$

where **YpI\_100\_1**, **YpI\_100\_2** and **YpI\_100\_p** are the **YpI\_100** values for the 2 wefts and the piercing warp.

The **L** calculated above is the *minimum* length of the repeat that will fit the two yarn systems (warp & weft) without requiring compression of the yarns.

Since we are analyzing the weft, the corresponding *maximum* PpI (Picks per Inch) is the number of wefts in the repeat divided by **L**. For the example just above this would be:

$$PpI = (N_{y1} + N_{y2}) / L \quad \text{where } N_{y1} \text{ and } N_{y2} \text{ are weft counts}$$

When analyzing the warp, **L** would be the length across the warp (along a weft) and the maximum EpI or Sett would be the number of warps in the repeat divided by **L**:

$$EpI = Sett = (N_{y1} + N_{y2}) / L \quad \text{where } N_{y1} \text{ and } N_{y2} \text{ are warp counts}$$

Example: weaving a plain weave using

weft: 8 NeC cotton, med soft twist (Fiber Fraction **FF**=0.50):  
calculated: Diam=0.01384, YpI\_100=72.2

warp: 14 NeC cotton, Reg warp twist (FF=0.60):  
calculated: Diam=0.00955, YpI\_100=104.7

**Ny** = 2 wefts/repeat; **Np** = 2 piercings/repeat

$$L = N_y \times D_y + N_p \times D_p = 2 \times 0.01384 + 2 \times 0.00955 = \mathbf{0.0468} \text{ " per repeat}$$

or, using **YpI\_100**:

$$L = N_y / Y_{pI\_100\_y} + N_p / Y_{pI\_100\_p} = 2 / 72.2 + 2 / 104.7 = 0.0468$$

The minimum weft spacing is the number of wefts per repeat / minimum length of repeat

$$PpI = N_y / L = 2 / 0.0468 = \mathbf{42.7} \text{ Picks per Inch.}$$

Compare that to the 5-shaft satin using the same yarns:

**Ny** = 5 wefts/repeat; **Np** = 2 piercings/repeat

$$L = N_y / Y_{pI\_100\_y} + N_p / Y_{pI\_100\_p} = 5 / 72.2 + 2 / 104.7 = 0.08835 \text{ " per repeat}$$

$$PpI = N_y / L = 5 / 0.08835 = \mathbf{56.6} \text{ Picks per Inch}$$



Similar calculations apply to the **Sett** or **EpI**, the difference being that the roles of the weft and warp are reversed:

$$N_y = \# \text{ of } \textit{warps}/\textit{repeat}; \quad N_p = \# \text{ of } \textit{piercings} \textit{ (by weft) } / \textit{repeat}$$

In the case of the plain weave we get the same Sett or EpI - because each repeat still has to accommodate 2 wefts plus 2 warps, whether analyzing along the weft or along the warp.

However for the 5-shaft satin:  $N_y = 5 \text{ warps}/\textit{repeat}; \quad N_p = 2 \text{ (weft) } \textit{piercings}/\textit{repeat}$

$$L = N_y / YpI_{100_y} + N_p / YpI_{100_p} = 5 / 104.7 + 2 / 72.2 = 0.0755 \text{ “ per repeat across warp}$$

$$EpI = N_y / L = 5 / 0.0755 = \mathbf{66.3 \text{ Ends per Inch}}$$
 which is denser than the **56.6 Picks per Inch**

The difference is that along the weft a repeat needs to provide space for 5 thinner warps plus 2 thicker weft piercings whereas along the warp each repeat must accommodate 5 thicker wefts plus 2 warp piercings, thus requiring more space.

The above geometric model is based on the assumption that the wefts all lie in the same plane so the piercing yarns run vertically through the plane.

This “all wefts in the same plane” is arguably realistic in the case of plain weaves where the position of the wefts is so tightly controlled by the warps. However weaves with longer floats provide more flexibility which may allow wefts to move up or down; this lets the warp make a gentler bend which pierces at an angle and permits a slightly increased density.



Ashenhurst offered a modified version of his rule to account for this effect which assumes that the weft and warp deflect each by their diameters in a “balanced” fabric with longer float lengths.

Looking at the figure you can see that the gap for the piercing warp is now on a diagonal; which means that in the yellow triangle, it is the diagonal “hypotenuse” which must be long enough to allow  $D_{warp} + D_{weft}$  while the vertical side =  $D_{weft}$ . The base is the weft-to-weft spacing across the transition; by Pythagorus’s Theorem:

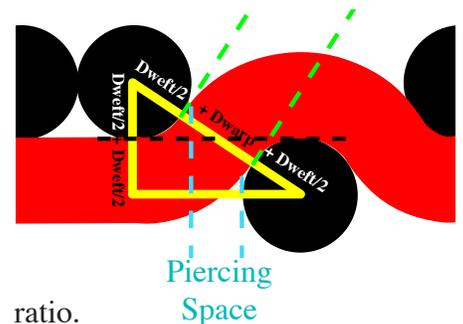
$$c^2 = a^2 + b^2 \quad \text{or} \quad b^2 = c^2 - a^2 \quad \text{or} \quad b = \textit{sqrt}[ c^2 - a^2 ]$$

so, using the 3rd equation, weft-to-weft spacing across transition =  $\textit{sqrt}[ (D_{warp} + D_{weft})^2 - D_{weft}^2 ]$

Of that amount,  $D_{weft}$  is used by the half-wefts on either side of the warp transition, so the extra space needed just for the warp’s transition is

$$\textit{Piercing space} = \textit{sqrt}[ (D_{warp} + D_{weft})^2 - D_{weft}^2 ] - D_{weft}$$

since it is purely a geometric issue the proportional effect will be determined by the **ratio** of the yarn diameters, not by specific yarn sizes. For example, a #10 weft with #40 warp and a #2 weft with #8 warp both have the same 1:4 ratio and thus identical geometry. This makes it possible to derive a formula which calculates the effects of the geometry based only on the yarn diameter ratio.



If we divide both by  $D_{warp}$  we get

$$D_{weft}/D_{warp} = R = \textit{ratio of weft diameter to warp diameter}$$

$$D_{warp}/D_{warp} = 1$$

Substituting this into the equation we get:

$$\textit{Piercing space}/D_{warp} = \textit{sqrt}[ (D_{warp}/D_{warp} + D_{weft}/D_{warp})^2 - (D_{weft}/D_{warp})^2 ] - D_{weft}/D_{warp}$$

$$\textit{Piercing space}/D_{warp} = \textit{sqrt}[ (1+R)^2 - (R)^2 ] - R = \textit{sqrt}[ 1 + 2R + R^2 - R^2 ] - R = \textit{sqrt}[ 1 + 2R ] - R$$

$$\textit{Piercing space}/D_{warp} = \textit{sqrt}[ 1 + 2R ] - R \quad \text{or} \quad \textit{Piercing space} = ( \textit{sqrt}[ 1 + 2R ] - R ) \times D_{warp}$$

Evaluating this for various Weft/Warp diameter ratios we get

Dweft/Dwarp = 0.500 Piercing space =  $\sqrt{1 + 2 \times 0.500} - 0.500 = \sqrt{2.000} - 0.500 = \mathbf{0.914} \times \text{Dwarp}$

Dweft/Dwarp = 0.707 Piercing space =  $\sqrt{1 + 2 \times 0.707} - 0.707 = \sqrt{2.414} - 0.707 = \mathbf{0.847} \times \text{Dwarp}$

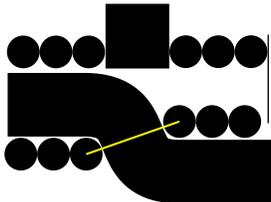
Dweft/Dwarp = 1.000 Piercing space =  $\sqrt{1 + 2 \times 1.000} - 1.000 = \sqrt{3.000} - 1.000 = \mathbf{0.732} \times \text{Dwarp}$

Dweft/Dwarp = 1.414 Piercing space =  $\sqrt{1 + 2 \times 1.414} - 1.414 = \sqrt{3.828} - 1.414 = \mathbf{0.542} \times \text{Dwarp}$

Dweft/Dwarp = 2.000 Piercing space =  $\sqrt{1 + 2 \times 2.000} - 2.000 = \sqrt{5.000} - 2.000 = \mathbf{0.236} \times \text{Dwarp}$

**Dweft/Dwarp = 2.414 or higher Piercing space = 0, no extra spacing being needed.**

Dweft/Dwarp = 0.5



Dweft/Dwarp = 1.0



Dweft/Dwarp = 2.0



Ashenhurst first rule spacing

Ashenhurst modified rule spacing

When the piercing warp is larger than the weft being pierced, the spacing does not reduce much because the weft offset is small and the spacing is dominated by the warp which runs almost perpendicular. However, when the warp is small compared to the weft, then the weft offset leaves a large gap through which the warp can pass on a shallow diagonal without requiring much if any extra weft spacing.

This leads us to a modified form of the repeat length equation

$L = N_{y1} \times D_{y1} + N_p \times \text{PierceF} \times D_p$ , where

$N_{y1}$ ,  $D_{y1}$  are the number and diameter of the wefts (in this case) in the repeat, and

$N_p$ ,  $D_p$  are the number and diameter of the piercing yarns (warp in this case) in the repeat, and

$\text{PierceF} = \sqrt{1 + 2R} - R$  where  $R = \text{Dweft/Dwarp}$ , i.e., the factor from above (0 if  $R \geq 2.414$ )

Looking at a 5-shaft satin using equal warp and weft diameters we have a total repeat length

5 wefts with 2 warp transitions

original width =  $5 D_{\text{weft}} + 2 D_{\text{warp}} = 7.00$

modified width =  $5 D_{\text{weft}} + 2 \times 0.732 D_{\text{warp}} = 6.46$

modified/original =  $6.46 / 7.00 = 0.92$ , or about a 11% ( $1/0.92$ ) increase in maximum thread density

And a 2/1 twill using equal warp and weft diameters will have a total repeat length

3 wefts with 2 warp transitions

original width =  $3 D_{\text{weft}} + 2 D_{\text{warp}} = 5.00$

modified width =  $3 D_{\text{weft}} + 2 \times 0.732 D_{\text{warp}} = 4.46$

modified/original =  $4.46 / 5.00 = 0.89$ , or about a 12% ( $1/0.89$ ) increase in maximum thread density

While a 2/1 twill using a warp that is 1/2 the weft diameter will have a total repeat length

3 wefts with 2 warp transitions

original width =  $3 D_{\text{weft}} + 2/2 D_{\text{warp}} = 4.00$

modified width =  $3 D_{\text{weft}} + 2/2 \times 0.0 D_{\text{warp}} = 3.00$

modified/original =  $3.0 / 4.00 = 0.75$ , or about a 33% ( $1/0.75$ ) increase in maximum thread density

In short: Ashenhurst's modified rule allows significant density increases when using thin warps (relative to the wefts) with weaves that have frequent piercings, i.e., short float lengths. The increases are much smaller when using larger warps or less frequent piercings.

Ashenhurst assumes flexible warps and wefts - which may not always be valid. For example a very rigid weft could force the warp to do all the bending, or a very taut warp could force the weft to do all the bending.

Note that the **EpI** or **PpI** calculated above are *maximum* yarn densities which result in yarns being in direct contact without squashing. Such fabrics tend to be stiff and most fabrics are woven with larger yarn spacings. There are numerous “rules of thumb” about how much to decrease the maximum density for different fabrics. Marg Coe, writing on WeaveTech, references one such guide, said to be “*derived from UK sources way back*”, which suggests using the following portions of the maximum:

- Furnishing Fabric 80-90%
- Clothing 60-80%
- Scarves & Shawls 50-60%

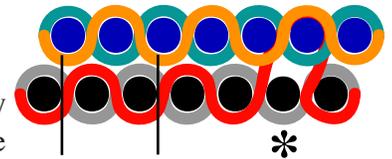
As always, weaving samples before starting production is the only certain approach...

## Exceptions and cautions:

A fundamental assumption underlying the above calculations is that all the warps and wefts should lie in a single layer. This is valid for many fabrics, but emphatically not the case for multi-layer fabrics and many other constructions.

For example, consider this 2-layer fabric, seen from the side (cut wefts):

While it would appear at first glance that each repeat includes 4 wefts, in fact only 2 wefts occupy each separate layer. The PpI calculation must be appropriate to the individual layers while the Pick Wheel (fabric advance) will be set to the sum of the individual layer PpI.



In other words, for each layer the Repeat **L** is calculated based only on the wefts and warps which pierce that layer, in this case  $N_y = 2$ ,  $N_p = 2$ . Since the layers are to be bound together, the *longest L* calculated for any layer determines the repeat length which must be used for all layers.

Individual layer PpI can then be calculated by dividing the number of wefts ( $N_y$ ) in each layer by that common repeat **L**, however this primarily of academic interest.

The Pick Wheel (fabric advance) must be set to accommodate all the wefts in all the layers. In other words, Pick Wheel (loom **PpI**) is

$$\text{PpI loom} = \text{sum of wefts in a repeat from all layers} / \text{L}$$

In addition there will be some “layer binders”, as shown at “\*”. If these are infrequent, i.e., the layers are loosely bound, they can be ignored. If they are frequent, as is done for tightly bound layers, you may have to include them in your calculations as yet another warp or weft which needs its own space within the repeat...

The common “tap” construction provides a more complicated exception.

This uses a 4-, 6-, 8, or even 12-color sequence in the warps. These are woven against 3 wefts - a thick light weft, a thick dark weft and a thin binder weft.



Figuring is done by selecting one (or more) of the warps to be featured on the top, over either the light or dark weft - in the example I am showing “blue over dark”, then “red over dark”. The binder weft’s purpose is to bind down the featured warps which otherwise would “float” loosely over the surface. The binder weft in turn is bound down into the underlying fabric (not shown in illustration).

The un-featured warps and weft weave the structural fabric underneath. In the illustration I have only shown 3 warps - blue, red and green, but all the other unfeatured warps would also be weaving against the light weft in the lower layer.

By inspection we can tell that the upper layer will require the longest repeat length, hence it will be the determining layer. Its **L** will be based on  $N_{y1}$  (fat weft) = 1,  $N_{y2}$  (binder weft) = 1,  $N_p$  (warp) = 2, i.e., a plain

weave using alternating weft thicknesses.

$$L = N_{y1} \times D_{y1} + N_{y2} \times D_{y2} + N_p \times D_p = D_{y1} + D_{y2} + 2 \times D_p, \text{ or}$$

$$L = 1/Y_{pI\_100\_1} + 1/Y_{pI\_100\_2} + 2/Y_{pI\_100\_d}$$

The loom PpI (Pick Wheel) will be

$$PpI \text{ loom} = 3 / L$$

because each length **L** of fabric contains 3 wefts: top layer's fat + binder wefts, plus the other unfeatured fat weft weaving in the underlying fabric layer.

Note that you may also have to estimate the "piercings" that occur as a pair of wefts switch surfaces, e.g., the light weft being brought to the surface and the dark weft being relegated to the back fabric. For a brief moment as the wefts switch layers the center of the fabric contains 2 fat wefts side-by-side. Very "busy" designs which swap wefts frequently may require slightly wider pick spacings than more conservative designs. These weft swaps also appear as "piercings" of the warp sheet, hence may also require a slightly lowered sett or EpI.

Similar considerations come into play with the "pick and pick" or "pick pick and pick" structures commonly used with upholstery fabrics. These usually involve a nondescript thin warp combined with 2- or 3-color weft sequences in which figuring is accomplished by pushing one (or two) of the wefts towards the top surface while submerging the other weft(s) towards the back surface. While not strictly a double-layer fabric, the wefts are stacked vertically, hence require less spacing than if they were only in a single plane.

A "pick and pick" fabric effectively has 1/2 the wefts on each surface while a "pick pick and pick" fabric has 2 of the 3 wefts in one surface and the 3rd weft in the other surface. Calculations of pick densities need to take this into account, i.e., calculate based on 1/2 of the wefts for "pick and pick" and 2/3 of the wefts for "pick pick and pick". As in the case of "tap" constructions, frequent weft swapping may require relaxed warp and weft spacings in active designs which frequently swap wefts.

Tom Dundin of Dundin Textile Designs pointed out that most fabrics include a variety of weaves, each of which can have its own optimum yarn densities - which adds another layer of complexities. For example: while a basic 8-shaft weft satin has 2 piercings per 8 yarns in both directions, a "shading satin" developed from it by shortening the weft floats to bring more warp to the face will result in 4 piercings by the warp per 8 wefts. If most of the fabric was composed of shading satins then it might be best to select the lower densities best suited for them and allow the areas of pure weft-satin to be a bit loose as a consequence.

In general one should review the various weaves being used to estimate the optimal densities for each, and then settle on compromise values which best accommodate that range.

In some cases an intentional mis-match is a key part of the design. For example a patch of tabby (plain weave) in a correctly spaced twill or satin fabric will "pucker" because its maximum densities are much lower than that of the host fabric. The plain weave has maximum piercings and consequently it cannot fit in the closer spacings set for the host twill or satin weaves, so it must bulge away from the surface to accommodate its increased surface area...

Chorus: calculate then weave samples and evaluate them before going into production...

## Coverage - warp cover, weft cover, total cover

A gauze fabric is characterized by a large amount of “open space” between the yarns, in other words, by low *coverage* by the yarns. When a warp is set at  $Y_{pi\_100}$ , by definition there will be 100% coverage; lesser densities will result in some open space between the yarns.

Coverage is of interest because it relates to a fabric’s ability to block light or wind, or conversely to allow air or sound to pass through easily.

There are 3 coverage values: warp coverage, weft coverage and the resulting total coverage.

**Cwarp** =  $E_{pI} / Y_{pI\_100}$  i.e., simply the actual  $E_{pI}$  or  $Sett$  divided by the density at 100% coverage

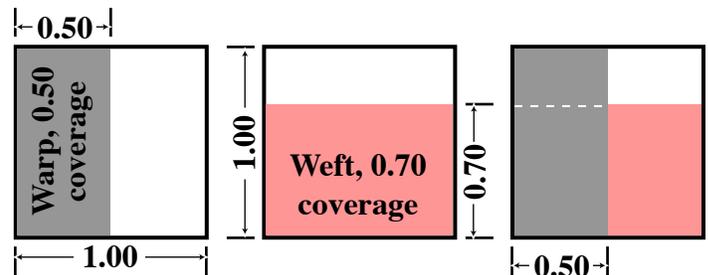
**Cweft** =  $P_{pI} / Y_{pI\_100}$

**Ctotal** = **Cwarp** + **Cweft** - (**Cwarp** x **Cweft**)

The formula for **Ctotal** can confuse. The figure to the right shows  $C_{warp} = 0.50$  and  $C_{weft} = 0.70$ , each as if all its yarns had been gathered together along one edge.

If you simply added the two you would have

$0.50 + 0.70 = 1.20$ , which obviously doesn’t make sense, especially as we can see the white area in the upper right corner which represents uncovered open areas.



The confusion occurs because the two coverage areas overlap, and you must not count the overlap twice! The area of overlap is a rectangle which is  $C_{warp}$  wide x  $C_{weft}$  tall - and this must be subtracted from the total.

Perhaps an easier way to conceptualize it is that if we take the entire warp area (gray,  $1.00 \times 0.50$ ) then we can only add that part of the weft area which is not overlapped, i.e., only the pink area in the right-hand panel.. This area would be  $0.70 \times (1.00 - 0.50)$ , or mathematically

**Ctotal** =  $1.00 \times C_{warp} + C_{weft} \times (1.00 - C_{warp}) = C_{warp} + C_{weft} - C_{warp} \times C_{weft}$  as above. QED.

In the above example we would calculate

$C_{total} = 0.50 + 0.70 - (0.50 \times 0.70) = 1.20 - 0.35 = \mathbf{0.85}$  coverage ( 85%)

Which, if the yarns were perfectly opaque, would suggest that 85% of light hitting the fabric would be blocked. Of course this would only be true with opaque monofilaments; normal yarns are much more complex and much less opaque... Nonetheless the **Ctotal** value does give a useful indication of porosity ( $1 - C_{total}$ ) and flexibility.

Calculating coverage for fabrics which mix yarn sizes follows the same logic. For example, if the weft alternated between thick and thin yarns you would simply calculate the weft coverage provided by each weft size and add them together to get the combined **Cweft**. An example:

$P_{pI} = 50$  wefts/inch, alternating between a #10 NeC ( $80.7 Y_{pI\_100}$ ) and a #20 NeC ( $114 Y_{pI\_100}$ ) cottons since the wefts are alternating, the  $P_{pI}$  of each is  $50/2 = \mathbf{25}$  for each weft size

$C_{weft\_1} = P_{pI} / Y_{pI\_100} = \mathbf{25} / 80.7 = 0.31$  coverage by the #10 NeC wefts

$C_{weft\_2} = P_{pI} / Y_{pI\_100} = \mathbf{25} / 114 = 0.22$  coverage by the #20 NeC wefts

**Cweft** =  $0.31 + 0.22 = \mathbf{0.53}$  ( 53%) total coverage by the two wefts

Mixed warp sizes are handled the same way. From that point onwards the same **Ctotal** formula is used.

Bear in mind that “coverage” is a very soft number. Different yarns will bloom to differing degrees, which will affect the openness of the fabric, and certainly its resistance to air passage. Any finishing operation which tends to “felt” the fabric will reduce the gaps and cause an apparent increase in “coverage”...

## Fabric Shrinkage

At the moment of its insertion, the weft is lying essentially straight and without any tension. The reed then beats it up into the fell (the edge of woven fabric) and the change in shed causes the warps to clamp it into place. At the fell the reed defines and maintains the fabric width. For a few inches beyond, the temples provide sideways tension to maintain fabric width. However past that point, and especially as the fabric gets past the sand roll, some shrinkage is almost always seen. Why?

If the wefts were perfectly rigid rods there would be no width shrinkage, but real yarns are not rigid rods. The warps apply pressure to the wefts, pushing them alternately one way, then the other way. Consequently the weft's originally straight path becomes curvy, which tends to draw its ends inwards, hence the shrinkage. The amount of shrinkage depends strongly on the weaves being used, among other factors.

While on the loom the warps are subject to constant tension which stretches them; once the woven fabric is past the sand roll that tension is removed and the fabric relaxes - causing lengthwise shrinkage.

When just removed from the loom, the fabric still contains residual tensions, held in place by friction. The finishing processes, such as washing, allow further relaxation thus further shrinkage.

Consequently there exist two sets of yarn density figures : EpI and PpI while on the loom, and higher EpI and PpI values in the fully finished fabric. The changes are almost never symmetric; the EpI might increase 5% while the PpI was increasing by 15%.

This brings up a subtle point: **if you want your figures to be a specific size or shape (e.g., a circle rather than an oval), then you need to use the EpI and PpI values from the finished fabric, not the on-the-loom values, in planning your design and setting your Aspect Ratio.**

A second point - the weight of your fabric is specified per square yard of **finished** fabric, in other words at the higher yarn densities.

## Warp Crimp

Warp Crimp describes the observation that the fabric produced is significantly shorter than the length of warp consumed. This is because the warp's path in the fabric is not straight. The degree of warp crimp depends strongly on the weaves being used - a plain weave causes many more wiggles, hence more crimp than would a satin weave. It is also affected by the size and nature of the wefts - a softer weft compresses more easily thus allowing the warp to lie straighter while a hard weft forces the warp to deflect around it and take a longer path. Higher warp tensions will tend to reduce crimp by squashing the wefts more. As with Sett, PpI, and coverage, the best that can be done is to estimate the effects; the only true test is to weave a sample.

Warp crimp is an important parameter because it directly affects the fabric's cost (amount of warp consumed per square yard of fabric) and the fabric's weight.

It also constrains the kinds of designs can be woven without causing imbalances in the warp tension - the "warp crimp balance" issue.

Consider a design with a border; say you wanted a plain-weave border around a floral design woven with 8-shaft satin shading weaves. In the floral (satin weaves) area the warp will change sides twice in every 8 picks, i.e., 2 deflections per 8 picks. In the plain-weave border the warp will be changing sides on every pick, 8 deflections per 8 Picks, 4X as often as the floral, thus taking a much longer pathway through the fabric.

This design has a severe warp crimp balance problem; fairly quickly the warps in the border will become much tighter than those in the floral area. If you are "weaving yardage", as in any commercial mill, one of two things will soon happen. If the border is wide enough that its warps have the strength to pull more warp off the beam, then the other warps will become slack, start to droop, and eventually either trip off the broken warp detectors or, worse, snarl with each other. If the border is narrower, then its warps will simply snap from the tension.

There really only two solutions:

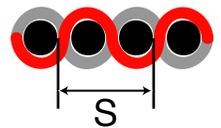
- 1) separate the warps onto 2 or more beams so the "busy" warps, i.e., those weaving the border, can be consumed faster than the less active areas. This is rarely practical - only when the entire beam's contents are to be woven in that single design.
- 2) change the design and weaves to avoid creating such imbalances - by far the more practical solution.

Warp crimp balance problems are managed by keeping an eye on the "activity" of your weaves, i.e., the frequency with which the warp changes sides, and avoiding major imbalances. For instance, be careful about mixing a high activity plain weave with low activity weaves, such as satins, in the same design.

Where your design requires significant differences between weaves, avoid long vertical sections of "outlier" (far from average activity) weaves and where the design requires them, consider using some compensating weaves as needed above and below to either reduce tension or take up slack as needed.

Smaller imbalances are absorbed by the elasticity of the warp, so elastic warps will be more tolerant while stiffer warps will demand more care. As always, the true test is to weave samples and watch the warp tension.

The worst case for high warp crimp is a tight plain weave, as at the right. The warp (red) is forced to "take the long path" around a rigid weft. In each repeat (**S**) the warp first follows the half-circle over a weft, then the half circle under the next weft. In other words over distance fabric length **S** (spanning 2 wefts) the warp is following the equivalent of a full circle path around one weft.



For example, assume the warp is 1/2 the diameter of the weft - so each repeat **S** contains 2 wefts of diameter **x** plus 2 warps of diameter 1/2 **x**, for a total of 3 **x**.

So Weft diameter =  $S/3$  and Warp diameter =  $S/6$ .

The center-line of the warp is following a circle whose diameter = Weft diameter plus the Warp diameter, i.e., =  $S/3 + S/6 = 2 S/6 + S/6 = 3S/6 = S / 2$

Circumference =  $\pi D = \pi (S / 2) =$  length of warp consumed in distance S along the fabric

Warp Crimp = length of warp / length of fabric =  $\pi (S / 2) / S = \pi (1/2) = 1.57$

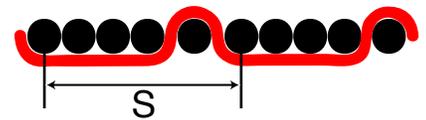
which is to say that warp consumption will be 57% greater than fabric production.

A useful concept for estimating warp crimp in other tight single layer weaves is that each time a warp changes sides it must wrap a 1/2 turn around a weft. In the plain weave above, the red warp does a 1/2 turn (by quarter-turns in an S shape) to move from under the centerline of the first weft up to above the centerline of the 2nd weft, then it does a second 1/2 turn in moving back down to under the centerline of the 3rd weft.

As discussed above, the centerline of the warp is bending around a circle whose diameter = is the weft diameter plus the warp diameter - because the warp centerline is 1/2 warp diameter outside of the weft's perimeter, thus adding 1/2 warp diameter to each side = 1 warp diameter total.

The warp length for each half turn =  $\pi D / 2 = \pi (D_{weft} + D_{warp}) / 2$

Consider this 5-shaft satin weave. Each repeat (S) contains 5 wefts plus 2 warp crossings. If the PpI has been calculated using *Ashenhurst's rule* so that there is just enough spacing to accommodate the 5 wefts and 2 warp piercings, the repeat length S will be:

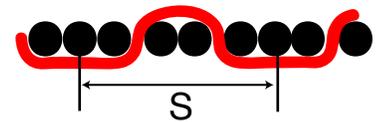


$$S = 5 \times D_{weft} + 2 \times D_{warp} = 5 / YpI_{100\_weft} + 2 / YpI_{100\_warp}$$

in that repeat the warp lies straight across 3 weft widths, but bends around 2. The total warp length will be

$$L = 3 D_{weft} + 2 \text{ half turns} = 3 D_{weft} + 2 ( \pi (D_{weft} + D_{warp}) / 2 )$$

Note that it is only the total number of wefts and the total number of piercings which matter. Both the weaves shown require 2 turns due to the two piercings.



This lets us develop a more general equation:

Given the following for a weave repeat:

Nwft = # of wefts

Np = # of piercings by a warp changing sides

$S = Nwft \times D_{weft} + Np \times D_{warp}$  (the *Ashenhurst rule's* suggested yarn density),

(which also =  $Nwft / PpI$  if your PpI exactly matches *Ashenhurst's* recommendation)

$$L = (Nwft - Np) \times D_{weft} + Np \times ( \pi (D_{weft} + D_{warp}) / 2 )$$

Warp Crimp  $WC = L / S$

$$WC = [ (Nwft - Np) \times D_{weft} + Np \times ( \pi (D_{weft} + D_{warp}) / 2 ) ] / [ Nwft \times D_{weft} + Np \times D_{warp} ]$$

since it is purely a geometric issue, the actual sizes don't matter - the Warp Crimp will be the same whether we are using #10 weft and #40 warp, or #2 weft and #8 warp; the important thing is that the diameter of the warp is 1/2 that of the weft. In other words, dividing the yarn diameters by the same amount won't change the geometric relationships.

If we divide both by  $D_{weft}$  we get

$D_{warp}/D_{weft} = R =$  ratio of warp diameter to weft diameter

$D_{weft}/D_{weft} = 1$

Substituting into the equation for warp crimp:

$$WC = [ (Nwft - Np) \times 1 + Np \times ( \pi (1 + R) / 2 ) ] / [ Nwft \times 1 + Np \times R ]$$

$$WC = [ Nwft + Np \times (0.5708 + 1.5708 R) ] / [ Nwft + Np \times R ] \quad \text{where } R = \text{Warp Diameter} / \text{Weft Diameter}$$

**Theoretical Warp Crimp for several weaves and warp/weft diameter ratios (at Ashenhurst PpI):**

weave =	plain	twill	twill	satin	satin	satin
	2x2	1/2	1/3	5-Sft	8-Sft	16-Sft
=====	=====	=====	=====	=====	=====	=====
# wefts	2	3	4	5	8	16
# piercings	2	2	2	2	2	2
Dwarp/Dweft=0.500	<b>1.57</b>	<b>1.43</b>	<b>1.34</b>	<b>1.29</b>	<b>1.19</b>	<b>1.10</b>
Dwarp/Dweft=0.707	<b>1.57</b>	<b>1.44</b>	<b>1.36</b>	<b>1.30</b>	<b>1.21</b>	<b>1.11</b>
Dwarp/Dweft=1.00	<b>1.57</b>	<b>1.46</b>	<b>1.38</b>	<b>1.33</b>	<b>1.23</b>	<b>1.13</b>

The above geometric model, satisfying as it may be, is based on two shaky assumptions: that the wefts all lie in the same plane and that the yarns are incompressible.

The “wefts in the same plane” is arguably accurate in the case of plain weaves where the position of the wefts is so tightly controlled by the warps. However weaves with longer floats provide more flexibility which may allow wefts to move up or down; this lets the warp make a gentler bend and also permits a slightly increased EpI or PpI.

Ashenhurst offered a modified version of his rule to account for this effect. Looking at the figure to the right you can see that the gap for the piercing warp is on a diagonal - it assumes that the weft and warp deflect each by their diameters in a “balanced” fabric.



Looking at the yellow triangle, it is the diagonal “hypotenuse” which must allow Dwarp + Dweft while the vertical side = Dweft. The base is the weft-to-weft spacing across the transition; by Pythagorus’s Theorem:

$$c^2 = a^2 + b^2 \quad \text{or} \quad b^2 = c^2 - a^2 \quad \text{or} \quad b = \text{sqrt}[c^2 - a^2]$$

$$\text{so weft-to-weft spacing across transition} = \text{sqrt}[(\text{Dwarp} + \text{Dweft})^2 - \text{Dweft}^2]$$

while Ashenhurst’s original rule specified Dwarp+Dweft, so comparing the two

$$\text{Ashenhurst modified} / \text{Ashenhurst original} = \text{sqrt}[(\text{Dwarp} + \text{Dweft})^2 - \text{Dweft}^2] / (\text{Dwarp} + \text{Dweft})$$

scaling as before by Dweft, with R = Dwarp/Dweft, simplifies this to :

$$\text{Ashenhurst modified} / \text{Ashenhurst original} = \text{sqrt}[R^2 + 2R] / (R + 1) \quad \text{which yields:}$$

$$\text{Dwarp/Dweft} = 0.500 \quad \text{modif/orig} = \text{sqrt}(1.25) / 1.5 = 0.75$$

$$\text{Dwarp/Dweft} = 0.707 \quad \text{modif/orig} = \text{sqrt}(1.91) / 1.707 = 0.81$$

$$\text{Dwarp/Dweft} = 1.000 \quad \text{modif/orig} = \text{sqrt}(3) / 2 = 0.87$$

Note that the above reductions are only for the weft-to-weft spacing across warp transitions.

Looking at a 5-shaft satin using equal warp and weft diameters we have a total repeat length

5 wefts with 2 warp transitions

$$\text{original width} = 5 \text{ Dweft} + 2 \text{ Dwarp} = 7.00$$

$$\text{modified width} = 5 \text{ Dweft} + 2 \times 0.87 \text{ Dwarp} = 6.74$$

$$\text{modified/original} = 6.74 / 7.00 = 0.96, \text{ or about a 4\% increase in thread density}$$

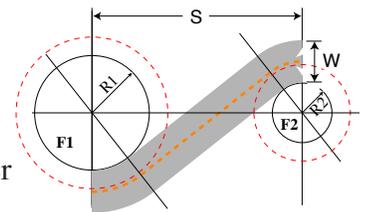
The “yarns are incompressible” may be accurate for incompressible monofilaments, but yarns are usually quite compressible, especially those used for wefts (50% air). Soft yarns will compress somewhat rather than forcing the warp to wrap in a perfect semi-circle around a rigid weft. Consequently the warp deflects less than calculated above, resulting in a lower warp crimp value. Similarly, the warp itself will compress, thus bringing its midline closer to the weft it is bending around, thereby further shortening its “long way around” path.

**Consequently these calculated warp crimp values are maximum worst-case values; practical fabrics will show lower crimp values.** According to Golec<sup>[1]</sup>, the typical range of warp crimps for single fabrics is **1.05** to **1.20**. Once again the weaving of test samples and careful measurement is the surest guide...

A further premise was that the weft PpI was calculated to provide just enough space for the wefts + piercing warps (*Ashenhurst’s rule*). In most cases the PpI is somewhat lower, and this reduces Warp Crimp in two ways. The piercing warps don’t have to “turn a right angle” as they pass through the weft sheet; instead they can take a more sloping path, which reduces the crimp. Secondly the extra spacing increases the fabric length per repeat which “dilutes” the effects of the warp’s deflection.

For this more general case, a single “warp bend” can be analyzed as follows: we have two wefts, **F1** and **F2**, spaced apart **S**. The warp is switching sides within this space and the question is how much extra warp is used up in making that switch.

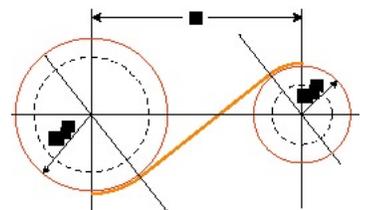
Looking at the figure we see that the warp’s length is composed of 3 segments: a straight middle section where the warp leaves contact with one weft and bridges over to the second weft, plus two arcs where it is in contact with a weft.



The weft’s thickness is represented by radii, **R1** and **R2**, each half of the corresponding weft diameters. The warp’s thickness is **W**. Note that the midline of the warp is shown by the orange dashed line; this represents the length of the warp with it compressing on the inside of a curve and stretching on the outside of a curve. The radii around which this centerline curves are thus **W/2** larger than the wefts (shown as dashed circles):

**Rf1 = R1 + W/2** and **Rf2 = R2 + W/2** as shown in this simplified 2nd figure:

This is the common “belt problem” for calculating reversing pulley belt lengths, see, for example, discussion at [http://en.wikipedia.org/wiki/Belt\\_problem](http://en.wikipedia.org/wiki/Belt_problem)



$$L_{\text{straight}} = \sqrt{S^2 - (Rf1 + Rf2)^2} \quad \text{the straight line portion}$$

$$L_{\text{arc}} = (Rf1 + Rf2) \arcsin((Rf1 + Rf2)/S) \quad \text{the 2 arcs, } \arcsin(x) = \sin^{-1}(x)$$

= the angle whose sine is x

$$L = L_{\text{arc}} + L_{\text{straight}} = A \arcsin(A/S) + \sqrt{S^2 - A^2} \quad \text{where } A = Rf1 + Rf2$$

or, calculating **A** in terms of the actual weft and warp diameters

$$A = Rf1 + Rf2 = R1 + W/2 + R2 + W/2 = R1 + R2 + W = D_{\text{weft1}} / 2 + D_{\text{weft2}} / 2 + D_{\text{warp}}$$

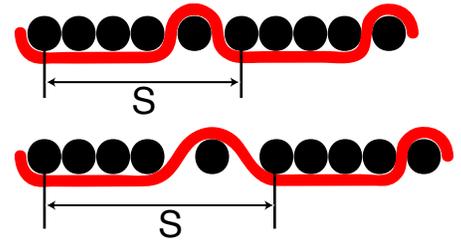
$$= (D_{\text{weft1}} + D_{\text{weft2}}) / 2 + D_{\text{warp}}$$

When both wefts are the same size, then the above reduces to **A = Dweft + Dwarp**

The Warp Crimp *for the warp bridging this single pair of wefts* = **L / S**

However, the Warp Crimp for the entire repeat must be calculated by adding the warp used for the sloped transitions described above to the warp which is lying straight over the rest of the repeat.

The extra spacing provided by a lower PpI is assumed to be entirely used by the warp piercing transitions. In other words, wefts lying next to each other still touch (why not) while the piercing warp spreads the wefts as much as possible - as shown in the 5-shaft satin figures to the right.



Our earlier calculation for a 5-shaft satin with equal diameter warp & weft diameters predicted a Warp Crimp of 1.33. It had a repeat width of 7 diameters (5 weft + 2 warp), and a warp length = 1.33 x 7 = 9.31.

Suppose that we decrease our PpI so as to include 9 diameters in the repeat length; i.e., to about 78% of max.

Previously each transition occurred in 2 diameters (from center of weft, across the piercing warp and to the center of the following weft). Now each transition has 3 diameters - the original 2 plus half of the extra space. From above, using actual diameters  $A = D_{\text{weft}} + D_{\text{warp}} = 2 \text{ Diameters} = 2 D$ , and transition width  $S = 3 D$

$$L_{\text{trans}} = 2 D \arcsin(2 D / 3 D) + \sqrt{(3 D)^2 - (2 D)^2} = 2 D \arcsin(2/3) + \sqrt{9 D^2 - 4 D^2}$$

$$= 2 D (0.73) + \sqrt{5 D^2} = 1.46 D + D \sqrt{5} = 1.46 D + 2.24 D = 3.7 D$$

$$L_{\text{warp}} = 3 \text{ straight portions} + 2 \text{ transitions} = 3 D + 2 (3.7 D) = 10.4 D$$

$$\text{Warp Crimp} = L_{\text{warp}} / \text{Repeat size} = 10.4 D / 9 D = 1.16$$

So reducing the PpI (by 22 %) has reduced the warp crimp from 1.33 to 1.16, about a 50% reduction in the extra warp taken up due to crimp.

**Summary:** Warp Crimp is highly variable so predictions from theoretical considerations can provide only a starting point. It is very important to record careful measurements of actual crimp in order to develop your own warp crimp “rules of thumb” appropriate to the specific constructions and yarns you use. The relevant factors are weaves, yarns, yarn densities and tension; change in any of them can change the warp crimp.

## Fabric Weight

Estimating the weight of the fabric is fairly straightforward, but requires an estimate of the warp crimp.

The information required is:

Warp Crimp, e.g., 1.25 (1.25 yards of warp per yard of fabric)

warp EpI and yarn measure

weft PpI and yarn measure

from which you calculate on-the-loom weight/Sq.yd as follows:

Convert both yarn measures into YpP (Yds/Lb)

Warp length per square yard of fabric = WarpCrimp x 36 x EpI

Weft length per square yard of fabric = 36 x PpI

Weight per square yard = Warp Length / Warp YpP + Weft Length / Weft YpP      *Lbs/Sq.Yd, on loom*

Example:

warp = #20 NeC cotton (YpP = 16,800) at 80 EpI with 1.25 warp crimp

weft = #10 NeC cotton (YpP = 8,400) at 75 PpI

warp length = 1.25 x 36 x 80 = 3,600 Yds per Sq.Yd

weft length = 36 x 75 = 2,700 Yds per Sq.Yd

weight per Sq.Yd = 3,600 / 16,800 + 2,700 / 8,400 = 0.214 + 0.321 = **0.535 Lbs/Sq.Yd** = 8.56 Oz/Sq.Yd

When a fabric contains a blend of different materials it is often necessary to calculate the % of each. This is done by calculating as above the weight of each component and dividing it by the total weight.

Footnotes:

[1] Golec, Edward L.; *Fundamentals of Woven Structures*; L.T.I. Bookstore, Lowell, MA; 1959

Changes made to original May 14, 2011, edition:

Apr 18, 2012: p.11 Dweft/Dwarp = 2.000 case was solved as 0.000 x Dwarp, corrected to **0.236 x Dwarp**; miscellaneous spelling corrections; added reference for Golec and a few clarifications.

October 25, 2012: p.5 rearranged equations into more symmetric forms; p.6 split K constants for clarity

November 14, 2012: corrected Golec spelling; edited some sentences to increase clarity.

March 20, 2013, v6: p.6 explained the physical meaning of K1 (diam in inches of #1 Ydens=1.00 yarn) and K2

April 7, 2014, v7: p.10 fixed typo changing “=  $\sqrt{1 + 2R + R^2 - R^2} - 1$ ” to “=  $\sqrt{1 + 2R + R^2 - R^2} - R$ ”

and clarified that spacing = 0 when  $R \geq 2.414$ . Thanks to Tom Dundin for spotting and reporting these errors!

Nov 3, 2014, v8: p.10 added figure, clarified text. p.13, J.2, corrected EpI to PpI